Int. J. Heat Mass Transfer. Vol. 19, pp. 1039–1043. Pergamon Press 1976. Printed in Great Britain

RANDOM ROUGH SURFACE MODEL FOR SPECTRAL DIRECTIONAL EMITTANCE OF ROUGH METAL SURFACES

R. C. BIRKEBAK
Department of Mechanical Engineering, University of Kentucky, Lexington, KY 40506, U.S.A.

and

A. ABDULKADIR
Consulting Engineer, Jakarta, Indonesia

(*Received* 8 *August* 1975 *and in revised form* 26 *January* 1976)

Abstract—The theoretical prediction of the spectral directional emittance of roughened metal surfaces are presented. The direct emission is evaluated by assuming the roughened surface to have a Gaussian random distribution of surface slopes and heights. The effect of random blockage of surface peaks by adjacent peaks in the direction of observation is taken into account.

NOMENCLATURE

A, area;
B, radiance;
F, shape factor; a point on elemental area;
$h(\mu - q_0)$, step function;
$\mathbf{i}, \mathbf{j}, \mathbf{k}$, unit vectors;
$L(\theta)$, $L_{b\lambda}$, spectral black body radiance;
m, standard deviation of the slopes;
N, normal vector to an elemental area;
$P(p)$, $P(q)$, and $P(\xi)$, Gaussian density functions defined by equations (13a–c);
p, q, r, local slopes at any points in the surface undulations;
S, $S(\theta)$, $S(F, \theta)$, and $S(\psi, \theta)$, blocking functions;
x, y, z, coordinates;
$Z(\theta)$, modified blocking function, defined by equation (25).

Greek symbols

α, angle, defined by equation (5);
β, sloping angle, $\beta = \tan^{-1} m$;
ε, emittance;
θ, observational angle, measured from normal of the test sample;
λ, wavelength;
Λ, $\Lambda(\mu)$, function defined by equation (15a);
μ, $\mu = \cot\theta$;
ξ, local height distribution;
σ, standard deviation of the heights distribution;
τ, angle, defined by equation (6);
$\bar{\tau}$, angle defined by equation (17);
Φ, radiant energy per time and unit surface area that passes through solid angle $d\omega$;
$\Delta\omega$, $d\omega$, solid angle.

Subscripts

b, blackbody;
m, mechanical; mean value; mean plane;
n, normal; the nth;
p, projected at the mean or horizontal plane;
r, reference; reflected; rough;
λ, spectral value.

INTRODUCTION

IT IS well known that roughened metal surfaces produce higher thermal emittance than the smooth surface of the same material. In general, the thermal emittance depends on the type of material used, the surface preparation of the samples, the surface temperature and the wavelength range involved. For roughened surfaces, the apparent emittance is the sum of the directly emitted energy from the surface asperities and the contribution of the multiple reflections inside the surface cavities. In this paper surfaces with slight surface undulations are considered and the contribution from the multiple reflections is neglected. In subsequent papers we will include multiple reflections in our analysis.

In attempts to predict the apparent emittance of roughened surfaces, several two-dimensional configurations have been suggested to approximate the topography of rough metal surfaces. V-grooves, parallel or circular grooves have been proposed to approximate the surface properties of, for instance, machine cut metal surfaces [1–4].

We have chosen a randomly roughened metal surface for our present analysis. Our proposed model combines random surface roughness and slopes and blockage effects. We have used as a basis of our work the results of Smith [5] whose model was developed for bi-directional reflectance calculations. In order to be able to use his results, the present authors have used Smith's terminology.

THE STATEMENT OF THE PROBLEM

Consider a plane surface upon which is superimposed positive and negative surface undulations generated by a stationary random process. The smooth surface can be taken as the mean plane and a statistical approach can be used to describe the distribution of the surface heights and slopes about the mean plane.

When thermal radiation is emitted by a very small area enclosing a point on a slope of the surface undulations, the probability that the radiant energy is detected by an instrument located at an angular position θ from the normal of the mean plane will depend upon the condition, that the energy will not be intercepted by any of the neighboring surface undulations or otherwise blockage will occur. When θ is zero or the detector views the sample from the normal position, no blockage occurs. As θ increases or the detector moves toward the grazing angle, there is an increase in the probability that blockage will occur. At a certain value of θ, complete blockage can be expected. This problem will necessitate the introduction of a blockage or shadowing function S, which will be discussed in the next section.

DIRECT ENERGY EMISSION

In order for us to obtain the emittance of a roughened surface we must first describe in detail the surface geometry. Consider a point F on an elemental area dA of any orientation, located on one of the surface undulations as shown in Fig. 1. The mean plane of the radiating area is A_m and **FZ** is the normal vector of this plane, **FN** is the local normal at point F and α is the angle between **FZ** and **FN**. τ is the angle between **FN** and the line of viewing **FB**, which is the axis of a solid angle dω which envelopes the radiated energy. The observation angle θ is the angle between **FZ** and **FB**, and is macroscopically measurable.

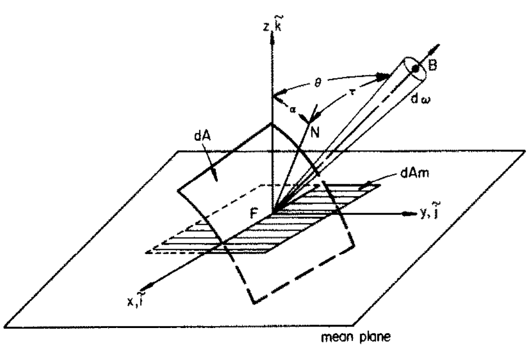

FIG. 1. Sketch of coordinate system.

The local slopes of F are defined as

$$p = \frac{\partial z}{\partial x}; \quad q = \frac{\partial z}{\partial y}; \quad r = \frac{\partial z}{\partial z} = 1. \tag{1}$$

From solid geometry,

$$\mathbf{FN} = p\mathbf{i} + q\mathbf{j} + \mathbf{k} \tag{2}$$

where **i**, **j** and **k** are the unit vectors in the directions of FX, FY and FZ, respectively. Also,

$$\mathbf{FB} = \sin\theta\mathbf{j} + \cos\theta\mathbf{k} \tag{3}$$

and

$$\mathbf{FZ} = \mathbf{k}. \tag{4}$$

Here, **FB** has been assumed to be located in the ZFY plane to eliminate the X-component in the further analysis. The angles α and τ are related to the slopes p, q and r by

$$\cos\alpha = \frac{\mathbf{FN}\cdot\mathbf{FZ}}{|\mathbf{FN}||\mathbf{FZ}|} = (1+p^2+q^2)^{-\frac{1}{2}} \tag{5}$$

and

$$\cos\tau = \frac{\mathbf{FN}\cdot\mathbf{FB}}{|\mathbf{FN}||\mathbf{FZ}|} = (q\sin\theta + \cos\theta)(1+p^2+q^2)^{-\frac{1}{2}}. \tag{6}$$

A detector located on **FB** looking at the radiating area A_m does not see the individual undulation of the radiating surface [5], but the projection of the mean plane perpendicular to the line of sight. Therefore, we can express the spectral energy emitted by the radiating area A_m in the direction θ within solid angle dω as

$$\mathrm{d}\Phi_{r\lambda} = \varepsilon_{r\lambda}(\theta) L_{b\lambda} A_m \cos\theta \, \mathrm{d}\omega \tag{7}$$

where $\varepsilon_{r\lambda}(\theta)$ is the apparent spectral directional emittance of the roughened surface and $L_{b\lambda}$ is the spectral distribution of the black-body radiance. In general, equation (7) accounts for the contribution of the multiple reflections, blockage, and direct emission.

From a microscopic standpoint, however, we can write the thermal energy emitted locally from point F on elemental area dA within solid angle dω in the direction θ as

$$\mathrm{d}^2\Phi_\lambda = \varepsilon_\lambda(\tau)\cos\tau L_{b\lambda} S(F,\theta)\,\mathrm{d}A\,\mathrm{d}\omega. \tag{8}$$

Here, we have taken into account the probability that the emitted thermal energy might be blocked by any neighboring undulation by introducing the local blocking function $S(F, \theta)$ of point F. The elemental area dA is assumed smooth, therefore, $\varepsilon_\lambda(\tau)$ is the spectral directional emittance of a smooth surface.

Further, we have assumed that dA is very small, and

$$\mathrm{d}A = \frac{\mathrm{d}A_m}{\cos\alpha} \tag{9}$$

where dA_m is the elemental area dA projected on the mean plane. The ratio of equations (6) and (5) is needed and is

$$\frac{\cos\tau}{\cos\alpha} = q\sin\theta + \cos\theta. \tag{10}$$

When equations (9) and (10) are substituted into (8) we obtain

$$\mathrm{d}^2\Phi_\lambda = \varepsilon_\lambda(\tau)[q\tan\theta + 1]L_{b\lambda}\cos\theta S(F,\theta)\,\mathrm{d}A_m\,\mathrm{d}\omega. \tag{11}$$

When we integrate over the entire radiating area A_m we have to take into account all possibilities of orientation of the surface slopes p and q as well as the distribution of heights ξ inside A_m. Therefore,

$$\mathrm{d}\Phi_\lambda = \int_{A_m}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon_\lambda(\tau)L_{b\lambda}[q\tan\theta + 1] \times \cos\theta S(F,\theta)P(p)P(q)P(\xi)\,\mathrm{d}\xi\,\mathrm{d}q\,\mathrm{d}p\,\mathrm{d}A_m\,\mathrm{d}\omega \tag{12}$$

where the surface density functions $P(p)$, $P(q)$ and $P(\xi)$ are assumed Gaussian:

$$P(p)=\frac{1}{m\sqrt{(2\pi)}}\exp\left[-\frac{p^2}{2m^2}\right] \quad (13a)$$

$$P(q)=\frac{1}{m\sqrt{(2\pi)}}\exp\left[-\frac{q^2}{2m^2}\right] \quad (13b)$$

$$P(\xi)=\frac{1}{\sigma\sqrt{(2\pi)}}\exp\left[-\frac{\xi^2}{2\sigma^2}\right] \quad (13c)$$

m and σ are the standard deviation of the slopes' and heights' distribution, respectively. The blocking function of point F, $S(F,\theta)$, is the probability that radiant energy emitted from an elemental area enclosing F will not be obstructed by any of its neighboring undulations. Smith [5] has derived and presents an expression for $S(F,\theta)$:

$$S(F,\theta)=S(p,q,\xi,\theta)=h(\mu-q)\left[1-\tfrac{1}{2}\operatorname{erfc}\left(\frac{\xi}{\sigma\sqrt{2}}\right)\right]^{\Lambda(\mu)} \quad (14)$$

where

$$\Lambda(\mu)=\tfrac{1}{2}\left[\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\frac{m}{\mu}\exp\left(-\frac{\mu^2}{2m^2}\right)-\operatorname{erfc}\left(\frac{\mu}{m\sqrt{2}}\right)\right] \quad (15a)$$

and

$$\mu=\cot\theta. \quad (15b)$$

The step function $h(\mu-q)$ is defined such that for a local observational angle $q<\cot\theta$, its value is unity and otherwise is zero for $q\geqslant\mu$. In deriving equations (14) and (15a), Smith has assumed a Gaussian distribution of slopes and heights.

Substituting equations (13a–c) and (14) into equation (12) we obtain

$$\mathrm{d}\Phi_\lambda=\frac{A_m}{m^2\sigma(2\pi)^{\frac{3}{2}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[1-\tfrac{1}{2}\operatorname{erfc}\left(\frac{\xi}{\sigma\sqrt{2}}\right)\right]^{\Lambda(\mu)} \times h(\mu-q)[q\tan\theta+1] \times \varepsilon_\lambda(\tau)L_{b\lambda}\cos\theta\exp\left[-\frac{p^2+q^2}{2m^2}-\frac{\xi^2}{2\sigma^2}\right] \times \mathrm{d}p\,\mathrm{d}q\,\mathrm{d}\xi\,\mathrm{d}\omega. \quad (16)$$

We can simplify equation (16) to an extent by examining the behavior of $\varepsilon_\lambda(\tau)$. τ is the angle between the local normal and the line of observation. Macroscopically, however, τ can be interpreted as the angle between the line of viewing and the normal of a hypothetical plane which has its orientation given by the $\tan^{-1}m$. This interpretation should be true when we consider that all local slopes contribute in a statistical way to the overall effect. Therefore, we introduce an angle $\bar{\tau}$ defined as

$$\bar{\tau}=\theta-\tan^{-1}m \quad (17)$$

and when $\bar{\tau}$ is substituted for τ, $\varepsilon_\lambda(\bar{\tau})$ can be removed from under the integrals. $\varepsilon_\lambda(\bar{\tau})$ is then evaluated from Fresnel relationships.

After integration with respect to p, simplification is made by expressing equation (16) as

$$\mathrm{d}\Phi_\lambda=\mathrm{d}\Phi_\lambda^{(1)}+\mathrm{d}\Phi_\lambda^{(2)} \quad (18)$$

with:

$$\mathrm{d}\Phi_\lambda^{(1)}=\frac{\varepsilon_\lambda(\bar{\tau})L_{b\lambda}A_m\cos\theta}{m\sigma(2\pi)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\mu-q) \times\left[1-\tfrac{1}{2}\operatorname{erfc}\left(\frac{\xi}{\sigma\sqrt{2}}\right)\right]^{\Lambda(\mu)} \times q\tan\theta\exp\left[-\frac{q^2}{2m^2}-\frac{\xi^2}{2\sigma^2}\right]\mathrm{d}q\,\mathrm{d}\xi\,\mathrm{d}\omega \quad (19)$$

and

$$\mathrm{d}\Phi_\lambda^{(2)}=\frac{\varepsilon_\lambda(\bar{\tau})L_{b\lambda}A_m\cos\theta}{m\sigma(2\pi)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\mu-q) \times\left[1-\tfrac{1}{2}\operatorname{erfc}\left(\frac{\xi}{\sigma\sqrt{2}}\right)\right]^{\Lambda(\mu)} \times\exp\left[-\frac{q^2}{2m^2}-\frac{\xi^2}{2\sigma^2}\right]\mathrm{d}q\,\mathrm{d}\xi\,\mathrm{d}\omega. \quad (20)$$

Smith [5] has obtained a solution for the integral part of equation (20) and using his result, one obtains

$$\mathrm{d}\Phi_\lambda^{(2)}=\varepsilon_\lambda(\bar{\tau})L_{b\lambda}A_m\cos\theta\frac{\left[1-\tfrac{1}{2}\operatorname{erfc}\left(\frac{\mu}{m\sqrt{2}}\right)\right]}{\Lambda(\mu)+1}\mathrm{d}\omega. \quad (21)$$

The integration with respect to the variable ξ in equation (19) is also given by Smith [5] without affecting q or θ:

$$\mathrm{d}\Phi_\lambda^{(1)}=\frac{\varepsilon_\lambda(\bar{\tau})L_{b\lambda}A_m\cos\theta}{m\sqrt{(2\pi)}}\int_{-\infty}^{\infty}\frac{h(\mu-q)}{\Lambda(\mu)+1} \times(q\tan\theta)\exp\left(-\frac{q^2}{2m^2}\right)\mathrm{d}q\,\mathrm{d}\omega. \quad (22)$$

By making use of the definition of the step function $h(\mu-q)$, the integral part of equation (22) gives

$$\int_{-\infty}^{\infty}\frac{h(\mu-q)}{\Lambda(\mu)+1}(q\tan\theta)\exp\left(-\frac{q^2}{2m^2}\right)\mathrm{d}q=\frac{\tan\theta}{\Lambda(\mu)+1}m^2\left[2-\exp\left(-\frac{\mu^2}{2m^2}\right)\right]. \quad (23)$$

By substituting equation (23) into (22) and when this result, with equation (21) are inserted into equation (18), we obtain the following expression

$$\mathrm{d}\Phi_\lambda=\varepsilon_\lambda(\bar{\tau})L_{b\lambda}Z(\theta)A_m\,\mathrm{d}\omega \quad (24)$$

where $Z(\theta)$ is the modified blocking function and is expressed as

$$Z(\theta)=\frac{1}{\Lambda(\mu)+1}\left[\frac{m}{\sqrt{(2\pi)}}\sin\theta\left(2-\exp\frac{-\mu^2}{2m^2}\right)+\cos\theta\left\{1-\tfrac{1}{2}\operatorname{erfc}\left(\frac{\mu^2}{m\sqrt{2}}\right)\right\}\right]. \quad (25)$$

The dependence of $Z(\theta)$ for various selected values of m is presented in Fig. 2. For θ close to zero, $Z(\theta)$ has a limiting value of unity for all m's. For larger values of θ, however, $Z(\theta)$ depends upon parameter m, and $Z(\theta)$ goes to zero when $\theta=90°$, for all m's. For $m\rightarrow 0$, we obtain

$$\lim_{m\rightarrow 0}Z(\theta)=\cos\theta \quad (26)$$

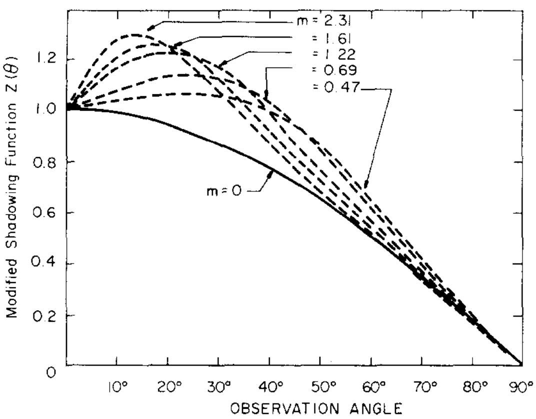

FIG. 2. Modified shadowing or blocking function for rough surfaces, $Z(\theta)$ vs θ.

which is physically true, since the area of smooth surface when observed from an angular position θ is equal to $\cos\theta$ times the magnitude of the surface area.

For very small values of m or if the surface is only very slightly undulated, the contribution from the multiple reflections can be neglected. The apparent spectral directional emittance can be expressed by setting equation (7) equal to equation (24), such that

$$\varepsilon_{r\lambda}(\theta)\cos\theta = \varepsilon_\lambda(\bar{\tau})Z(\theta). \quad (27)$$

Notice, that for $m = 0$, $Z(\theta)$ becomes $\cos\theta$ and $\bar{\tau}$ becomes θ by means of equation (17). For smooth surface, $\varepsilon_{r\lambda}(\theta)$ is therefore compatible with $\varepsilon_\lambda(\bar{\tau})$.

RESULTS

The dependence of $\varepsilon_{r\lambda}(\theta)\cos\theta$ on the angular position θ is presented in Fig. 3 for gold and in Fig. 4 for chrome. For gold, the optical constants are taken from Schrocken [6] and for chrome, those of Lenham and

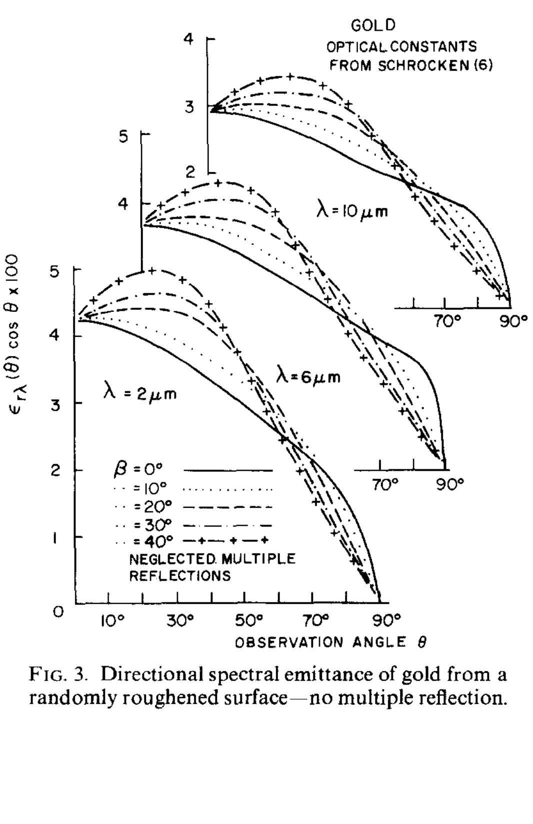

FIG. 3. Directional spectral emittance of gold from a randomly roughened surface—no multiple reflection.

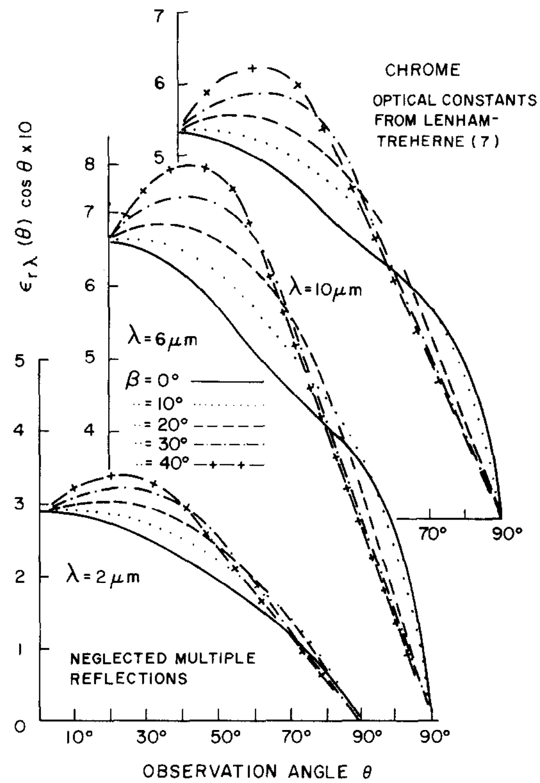

FIG. 4. Directional spectral emittance of chrome from a randomly roughened surface—no multiple reflection.

Treherne [7] are used. The results are presented for $\beta = \tan^{-1}m$ from 0 to 40° and for several selected wavelength values. Additional results are found in [8].

Certain general trends are evident in Figs. 3 and 4. The general shape of the curves are influenced greatly by the modified blocking function $Z(\theta)$. For angles of observation θ less than approximately 60°, the emittance increases with increasing surface roughness. When the observation angle θ exceeds 60°, the emittance of a roughened surface is smaller than a smooth surface of the same materials and the emittance decreases with increasing surface roughness. This decrease in directional emittance for large θ is a combined effect of surface geometry and smooth surface directional emittance.

REFERENCES

1. J. F. Ody-Sacadura, Influence de la rugosité sur le rayonnement thermique émis par les surfaces opaques: essai de modèle, *Int. J. Heat Mass Transfer* **15**, 1451–1465 (1972).
2. K. Kanayama, Apparent directional emittance of V-groove and circular-groove rough surfaces, *Heat Transfer Japan. Res.* **1**(1), 11–22 (1972).
3. E. M. Sparrow and J. L. Gregg, Radiant emission from a parallel-walled groove, *J. Heat Transfer* **C84**, 270–271 (1962).
4. E. M. Sparrow and S. H. Lin, Absorption of thermal radiation in a V-groove cavity, *Int. J. Heat Mass Transfer* **5**, 111–115 (1962).
5. B. G. Smith, Geometrical shadowing of a random rough surface, *IEEE Trans. on Antennas and Propagation* **AP-15**(5), 668–671 (September 1967).

6. K. Schrocken, Optical properties of satellite materials—the theory of optical and infrared properties of metals, NASA-TN 1523 (March 1963).
7. A. P. Lenham and D. M. Treherne, The optical properties of transition metals, in *Optical Properties and Electronic Structure of Metals and Alloys*, edited by F. Abeles. North Holland, Amsterdam (1966).
8. A. Abdulkadir, Spectral directional emittance of roughened metal surfaces, Ph.D. Dissertation, Univ. of Kentucky (1973).

MODELE DE SURFACE RUGUEUSE ALEATOIRE POUR L'ETUDE DE LA LUMINANCE MONOCHROMATIQUE DIRECTIONNELLE DE SURFACES METALLIQUES RUGUEUSES

Résumé—On présente une méthode de prévision théorique de la luminance monochromatique directionnelle de surfaces métalliques rugueuses. L'émission directe est évaluée en supposant que la surface rugueuse présente une distribution aléatoire gaussienne de pentes et de sommets. On prend en compte l'effet d'occultation aléatoire des pics par les pics adjacents, dans la direction d'observation.

MODELL DER RAUHIGKEITSVERTEILUNG ZUR BESTIMMUNG DES SPEKTRALEN EMISSIONSVERHÄLTNISSES UND DESSEN RICHTUNGSVERTEILUNG FÜR RAUHE METALLOBERFLÄCHEN

Zusammenfassung—Es wird über die theoretische Bestimmung des spektralen Emissionsverhältnisses und dessen Richtungsverteilung für aufgerauhte Metalloberflächen berichtet. Die direkte Strahlung wird ermittelt, in dem man für die Oberflächenrauhigkeiten eine Gauss'sche Verteilungsfunktion annimmt. Der Einfluss der gegenseitigen Abdeckung von Rauhigkeitserhebungen in Beobachtungsrichtung wird dabei berücksichtigt.

МОДЕЛЬ ПРОИЗВОЛЬНО-ШЕРОХОВАТОЙ ПОВЕРХНОСТИ ДЛЯ СПЕКТРАЛЬНОГО НАПРАВЛЕННОГО ИЗЛУЧЕНИЯ ШЕРОХОВАТЫХ МЕТАЛЛИЧЕСКИХ ПОВЕРХНОСТЕЙ

Аннотация — Представлены результаты теоретических расчетов спектрального направленного излучения шероховатых металлических поверхностей. Прямое излучение определяется при предположении, что шероховатая поверхность имеет гауссово случайное распределение углублений и возвышений на поверхности. Учитывается эффект случайного блокирования пиков на поверхности смежными пиками в направлении наблюдения.